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Discrete

$\pi \in (0,1)$	$Y \sim \text{Ber}(\pi)$ y = success/failure	$P(Y=y)=\pi^y(1-\pi)^{1-y}$ $y \in \{0,1\}$	$\text{E}[Y]=\pi$	$\text{Var}[Y]=\pi(1-\pi)$	$M_Y(t)=\pi e^t+(1-\pi)$
$\pi \in (0,1)$	$Y \sim \text{Bin}(m,\pi)$ y = successes in m trials	$P(Y=y)=\binom{m}{y}\pi^y(1-\pi)^{m-y}$ $y \in \{0,1,...,m\}$	$\text{E}[Y]=m\pi$	$\text{Var}[Y]=m\pi(1-\pi)$	$M_Y(t)=[\pi e^t+(1-\pi)]^m$
$\pi \in (0,1)$	$mY \sim \text{Bin}(m,\pi)$ my = successes in m trials	$P(Y=y)=\binom{m}{my}\pi^{my}(1-\pi)^{m-my}$ $my \in \{0,1,...,m\}$	$\text{E}[Y]=\pi$ $\text{E}[mY]=m\pi$	$\text{Var}[Y]=\frac{\pi(1-\pi)}{m}$ $\text{Var}[mY]=m\pi(1-\pi)$	$M_{mY}(t)=[\pi e^t+(1-\pi)]^m$
$\pi_j \in (0,1)\ \forall j$ s.t. $\sum_{j=1}^{k}\pi_j=1$	$\boldsymbol{Y} \sim \text{Multinom}(m,\boldsymbol{\pi})$ y_j = successes in j^{th} category	$P(\boldsymbol{Y}=\boldsymbol{y})=\binom{m}{\boldsymbol{y}}\pi_1^{y_1}\pi_2^{y_2}...\pi_k^{y_k}$ $y_j \in \{0,1,...,m\}\ \forall j$ s.t $\sum_{j=1}^{k}y_j=m$	$\text{E}[Y_j]=m\pi_j$	$\text{Var}[Y_j]=m\pi_j(1-\pi_j)$ $\text{Cov}[Y_i,Y_j]=-m\pi_i\pi_j, i\neq j$	$M_{\boldsymbol{Y}}(t)=\left[\sum_{j=1}^{k}\pi_j e^{t_j}\right]^m$
$\mu>0$ (rate) (expected occurrences)	$Y \sim \text{Poiss}(\mu)$ y = occurrences in a unit time	$P(Y=y)=\frac{e^{-\mu}\mu^y}{y!}$ $y \in \{0,1,2,...\}$	$\text{E}[Y]=\mu$	$\text{Var}[Y]=\mu$	$M_Y(t)=e^{\mu(e^t-1)}$
$\pi \in (0,1)$	$Y \sim \text{geom}(\pi)$ y = trials until 1 success	$P(Y=y)=\pi^1(1-\pi)^{y-1}$ $y \in \{1,2,3,...\}$	$\text{E}[Y]=\frac{1}{\pi}$	$\text{Var}[Y]=\frac{1-\pi}{\pi^2}$	$M_Y(t)=\frac{\pi e^t}{1-(1-\pi)e^t}$
$\pi \in (0,1)$	$Y \sim \text{NegBin}(r,\pi)$ y = trials until r successes	$P(Y=y)=\binom{y-1}{r-1}\pi^r(1-\pi)^{y-r}$ $y \in \{r,r+1,r+2,...\}$	$\text{E}[Y]=r\frac{1}{\pi}$	$\text{Var}[Y]=r\frac{1-\pi}{\pi^2}$	$M_Y(t)=\left[\frac{\pi e^t}{1-(1-\pi)e^t}\right]^r$
$\pi \in (0,1)$	$Y \sim \text{geom}(\pi)$ y = failures until 1 success	$P(Y=y)=\pi^1(1-\pi)^{y}$ $y \in \{0,1,2,...\}$	$\text{E}[Y]=\frac{1-\pi}{\pi}$	$\text{Var}[Y]=\frac{1-\pi}{\pi^2}$	$M_Y(t)=\frac{\pi}{1-(1-\pi)e^t}$
$\pi \in (0,1)$	$Y \sim \text{NegBin}(r,\pi)$ y = failures until r successes	$P(Y=y)=\binom{y+r-1}{y}\pi^r(1-\pi)^{y}$ $y \in \{0,1,2,...\}$	$\text{E}[Y]=r\frac{1-\pi}{\pi}$	$\text{Var}[Y]=r\frac{1-\pi}{\pi^2}$	$M_Y(t)=\left[\frac{\pi}{1-(1-\pi)e^t}\right]^r$
$N=0,1,2,...$ (Populat.) $K=0,1,...,N$ (Type I) $n=0,1,...,N$ (Sample)	$Y \sim \text{Hypergeom}(N,K,n)$ y = Type I objects in sample *sample drawn w/o replacement	$P(Y=y)=\frac{\binom{K}{y}\binom{N-K}{n-y}}{\binom{N}{n}}$	$\text{E}[Y]=n\frac{K}{N}$	$\text{Var}[Y]=n\frac{K}{N}\frac{N-K}{N}\frac{N-n}{N-1}$	$M_Y(t)$ – google/wiki

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Continuous 1

$a, b \in \mathbb{R}$	$Y \sim \text{Unif}(a,b)$	$f(y\|a,b) = \frac{1}{b-a}\mathbb{1}_{(a,b)}(y)$	$\text{E}[Y] = \frac{a+b}{2}$	$\text{Var}[Y] = \frac{(b-a)^2}{12}$	$M_Y(t) = \frac{e^{tb}-e^{ta}}{t(b-a)},\ \text{if } t \neq 0$ $= 1,\ \text{if } t = 0$
$\pi > 0$	$Y \sim \text{Unif}(0,\pi)$	$f(y\|\pi) = \frac{1}{\pi}\mathbb{1}_{(0,\pi)}(y)$	$\text{E}[Y] = \frac{\pi}{2}$	$\text{Var}[Y] = \frac{\pi^2}{12}$	$M_Y(t) = \frac{e^{t\pi}-1}{t\pi},\ \text{if } t \neq 0$ $= 1,\ \text{if } t = 0$
$\mu > 0$ (scale)	$Y \sim \text{Exp}(\mu)$ $Y \sim \text{Gamma}(1,\mu)$	$f(y\|\mu) = \frac{1}{\mu}e^{-\frac{1}{\mu}y}$ $y > 0$	$\text{E}[Y] = \mu$	$\text{Var}[Y] = \mu^2$	$M_Y(t) = (1-\mu t)^{-1}$ for $t < \frac{1}{\mu}$
$\alpha > 0$ (shape) $\beta > 0$ (scale)	$Y \sim \text{Gamma}(\alpha,\beta)$	$f(y\|\alpha,\beta) = \frac{1}{\Gamma(\alpha)\beta^\alpha}y^{\alpha-1}e^{-\frac{y}{\beta}}$ $y > 0$	$\text{E}[Y] = \alpha\beta$	$\text{Var}[Y] = \alpha\beta^2$	$M_Y(t) = (1-\beta t)^{-\alpha}$ for $t < \frac{1}{\beta}$
$\mu > 0$ $\left(\text{rate} = \text{scale}^{-1}\right)$	$Y \sim \text{Exp}(\mu)$ $Y \sim \text{Gamma}(1,\mu)$	$f(y\|\mu) = \mu e^{-\mu y}$ $y > 0$	$\text{E}[Y] = \frac{1}{\mu}$	$\text{Var}[Y] = \frac{1}{\mu^2}$	$M_Y(t) = \left(1-\frac{t}{\mu}\right)^{-1}$ for $t < \mu$
$\alpha > 0$ (shape) $\beta > 0$ $\left(\text{rate} = \text{scale}^{-1}\right)$	$Y \sim \text{Gamma}(\alpha,\beta)$	$f(y\|\alpha,\beta) = \frac{1}{\Gamma(\alpha)\beta^{-\alpha}}y^{\alpha-1}e^{-\beta y}$ $y > 0$	$\text{E}[Y] = \frac{\alpha}{\beta}$	$\text{Var}[Y] = \frac{\alpha}{\beta^2}$	$M_Y(t) = \left(1-\frac{t}{\beta}\right)^{-\alpha}$ for $t < \beta$
$\alpha > 0$ (shape) $\beta > 0$ (shape)	$Y \sim \text{Beta}(\alpha,\beta)$	$f(y\|\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}y^{\alpha-1}(1-y)^{\beta-1}$ $y \in (0,1)$	$\text{E}[Y] = \frac{\alpha}{\alpha+\beta}$ $\text{E}[Y^r] = \frac{\Gamma(\alpha+r)}{\Gamma(\alpha)}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha+\beta+r)}$	$\text{Var}[Y] = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	$M_Y(t) = 1 + \sum_{i=1}^{\infty}\left(\prod_{r=0}^{i-1}\frac{\alpha+r}{\alpha+\beta+r}\right)\frac{t^i}{i!}$
$\alpha_j > 0\ \forall j$	$\boldsymbol{\pi} \sim \text{Dirich}(\boldsymbol{\alpha})$	$f(\boldsymbol{\pi}\|\boldsymbol{\alpha}) = \frac{\Gamma(\alpha_1+...+\alpha_k)}{\Gamma(\alpha_1)...\Gamma(\alpha_k)}\pi_1^{\alpha_1-1}...\pi_k^{\alpha_k-1}$ $\pi_j \in (0,1)\ \forall j \quad \text{s.t.} \quad \sum_{j=1}^{k}\pi_j = 1$	$\text{E}[\pi_j] = \frac{\alpha_j}{\sum_{l=1}^{k}\alpha_l}$	$\text{Var}[\pi_j] = \frac{\alpha_j\left(\sum_{l=1}^{k}\alpha_l - \alpha_j\right)}{\left(\sum_{l=1}^{k}\alpha_l\right)^2\left(\sum_{l=1}^{k}\alpha_l + 1\right)}$	$M_{\boldsymbol{\pi}}(t)$ – google

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Continuous 2

$\mu \in \mathbb{R}$ (mean) $\sigma > 0$ (std. dev.)	$Y \sim \mathrm{N}(\mu, \sigma^2)$	$f(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(y-\mu)^2}$ $y \in \mathbb{R}$	$\mathrm{E}[Y] = \mu$	$\mathrm{Var}[Y] = \sigma^2$	$M_Y(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$
$\boldsymbol{\mu} \in \mathbb{R}^p$ (location) $\boldsymbol{\Sigma} \in \mathbb{R}^{p \times p}$ (P.D. Cov.)	$\boldsymbol{Y} \sim \mathrm{MVN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$	$f(y) = (2\pi)^{-\frac{k}{2}} \lvert\boldsymbol{\Sigma}\rvert^{-\frac{1}{2}} e^{-\frac{1}{2}(\boldsymbol{y}-\boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{y}-\boldsymbol{\mu})}$ $\boldsymbol{y} \in \boldsymbol{\mu} + \mathrm{span}(\boldsymbol{\Sigma}) \subseteq \mathbb{R}^p$	$\mathrm{E}[\boldsymbol{Y}] = \boldsymbol{\mu}$	$\boldsymbol{\Sigma}$	$M_{\boldsymbol{Y}}(t) = e^{\boldsymbol{t}'\boldsymbol{\mu} + \frac{1}{2}\boldsymbol{t}'\boldsymbol{\Sigma}\boldsymbol{t}}$
$p \in \mathbb{N}^+$ (d.f.)	$U \sim \chi^2_p$ $U \sim \mathrm{Gamma}(\frac{p}{2}, 2)$	$f(u) = \frac{1}{\Gamma(\frac{p}{2}) 2^{\frac{p}{2}}} u^{\frac{p}{2}-1} e^{-\frac{u}{2}}$ $u > 0$	$\mathrm{E}[U] = p$	$\mathrm{Var}[U] = 2p$	$M_U(t) = (1-2t)^{-\frac{p}{2}}$ for $t < \frac{1}{2}$
$p > 0$ (d.f.) $\phi > 0$ (noncentrality)	$U \sim \chi^2_p(\phi)$, where $U \mid J = j \sim \chi^2_{p+2j}$ $J \sim \mathrm{Pois}(\phi)$	$f(u) = \sum_{j=0}^{\infty} f(j) f(u \mid j)$ $= \sum_{j=0}^{\infty} \left[\frac{e^{-\phi}\phi^j}{j!}\right] \left[\frac{u^{\frac{p+2j}{2}-1} e^{-\frac{u}{2}}}{\Gamma(\frac{p+2j}{2}) 2^{\frac{p+2j}{2}}}\right]$ $u > 0$	$\mathrm{E}[U] = p + 2\phi$	$\mathrm{Var}[U] = 2p + 8\phi$	$M_U(t) = (1-2t)^{-\frac{p}{2}} e^{\frac{2t}{1-2t}\phi}$
$p_1 > 0$ (d.f.) $p_2 > 0$ (d.f.)	$W \sim \mathrm{F}_{p_1, p_2}$ $W = \frac{U_1/p_1}{U_2/p_2}$, where $U_1 \sim \chi^2_{p_1}$ $U_2 \sim \chi^2_{p_2}$	$f(w)$ – google/wiki $w > 0$	$\mathrm{E}[W]$ – google/wiki	$\mathrm{Var}[W]$ – google/wiki	$M_W(t)$ DNE
$p_1 > 0$ (d.f.) $p_2 > 0$ (d.f.) $\phi > 0$ (noncentrality)	$W \sim \mathrm{F}_{p_1, p_2}(\phi)$ $W = \frac{U_1/p_1}{U_2/p_2}$, where $U_1 \sim \chi^2_{p_1}(\phi)$ $U_2 \sim \chi^2_{p_2}$	$f(w)$ – google/wiki $w > 0$	$\mathrm{E}[W]$ – google/wiki	$\mathrm{Var}[W]$ – google/wiki	$M_W(t)$ DNE
$p > 0$ (d.f.)	$X \sim \mathrm{t}_p$ $X = \frac{Z}{\sqrt{U/p}}$, where $Z \sim \mathrm{N}(0,1)$ $U \sim \chi^2_p$	$f(x) = \frac{\Gamma\left(\frac{p+1}{2}\right)}{\sqrt{p\pi}\,\Gamma(\frac{p}{2})} \left(1 + \frac{x^2}{p}\right)^{-\frac{p+1}{2}}$ $x \in \mathbb{R}$	$\mathrm{E}[X] = 0$, if $p > 1$ $=$ DNE, o.w.	$\mathrm{Var}[X] = \frac{p}{p-2}$, if $p > 2$ $= \infty$, if $1 < p \leq 2$ $=$ DNE, o.w.	$M_X(t)$ DNE
$p > 0$ (d.f.) $\mu > 0$ (noncentrality)	$X \sim \mathrm{t}_p(\mu)$ $X = \frac{Y}{\sqrt{U/p}}$, where $Y \sim \mathrm{N}(\mu, 1)$ $U \sim \chi^2_p$	$f(x)$ – google/wiki $x \in \mathbb{R}$	$\mathrm{E}[X]$ – google/wiki	$\mathrm{Var}[X]$ – google/wiki	$M_X(t)$ DNE